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The Effects of Quantum Entropy on the Bag Constant

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Abstract

The effects of quantum entropy on the bag constant are studied at low temperatures and small chemical potentials. The inclusion of the quantum entropy of the quarks in the equation of state provides the hadronic bag with an additional heat which causes a decrease in the effective latent heat inside the bag. We have considered two types of baryonic bags, Δ and Ω^-. In both cases we have found that the bag constant without the quantum entropy almost does not change with the temperature and the quark chemical potential. The contribution from the quantum entropy to the equation of state clearly decreases the value of the bag constant.

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1 Introduction

The entropy is a concept which has taken on many meanings throughout the sciences. Its usual sense relates the heat changes to the likelihood of the related processes at various determined temperatures. In the limit of low temperatures Planck [1] noted that a mixture of different substances retained a finite entropy even at absolute zero. This result is quite contrary to the usual interpretation of the Nernst heat theorem, for which the entropy should vanish in the low temperature limit. It was Schrödinger [2] who pointed out a similar observation for N atoms each with a two level ground state resulting in 2^N states. In this case there should be an entropy of $N\ln 2$ with Boltzmann's constant k taken to be unity. These results are consistent with the quantum statistical

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definition [3] of the entropy S using the density matrix ρ which relates directly to the wavefunction. This entropy is given by the trace over the quantum states as follows:

$$S = -\ \mathrm{Tr}\ \left[\rho \ln \rho\right]. \tag{1}$$

Schrödinger's result [2] can be readily attained for a system of N spin one half states. Although the name "Quantum Entropy" implies the construction of the density matrix ρ from the quantum states, the actual mathematical form is well rooted in the laws of classical physics [1]. The entropy of mixing of different types of ideal gases can be calculated in the same way as Eq. (1) by replacing ρ with x_i, which is just the proportion of each constituent type i in the total gas system. Thus the entropy of mixing becomes $-\sum_i x_i \ln x_i$, where the sum has replaced the trace operation. This expression is clearly a constant independent of the temperature so that it must remain at absolute zero [1].

In recent work we have applied these ideas to the quark singlet ground state of the hadrons [4]. The color symmetry $SU(3)_c$ provides a quantum entropy for each of the colored quarks with the value $\ln 3$. The ground state entropy reflects the mixing probabilities inside the hadrons at vanishing temperature. We have extended this result to models at finite temperatures [5]. In this work we investigate the contribution of this entropy to the equation of state for the quarks inside the hadrons using the phenomenological bag model for strong interactions [6]. We assume that inside the hadron bag all of the strong interactions at low temperature T and small quark chemical potential μ_q are included in the quark and gluon condensates. In particular, we will look at a special model for baryons at low temperatures and small chemical potentials, for which the effective degrees of freedom are the quark and gluon colors. All other couplings are taken so that the spin and flavor are not explicitly considered.

In the limit of low temperatures the interactions in the ground state are expressed in terms of the vacuum expectation values of the quark and the gluon fields. The calculation of these vacuum contributions are gotten from the operator product expansion using the QCD sum rules [7, 8], which has the local operators of dimension four yielding the main contributions to the thermodynamics [9, 10, 11]. The pure gluon vacuum expectation value is calculated [12] from the product of the field strength tensors $G^a_{\mu\nu}G^{\mu\nu}_a$ including the renormalization group beta function, where the repeated indices are summed over their range of values. From hereon we shall simply write it as $< G^2 >_0$, which is often called the vacuum gluon condensate. This vacuum gluon condensate can be extracted [7] from the charmonium spectrum to yield a consistently estimated [6] value of about 1.95 GeV/fm^3. For the quark condensates $m_q < \bar{q}q >_0$ we consider two extreme cases - the light quarks $m_{lq} < u\bar{u} + d\bar{d} >_0$ and the pure strange quarks, $m_s < \bar{s}s >_0$, vacuum expectation values. We use the accepted value of the pion decay constant F_π as 92.4 MeV [6] with the respective average light quark mass m_{lq} of 6 MeV and strange quark mass m_s of 150 MeV. With these values together with the values of the average pion 138 MeV and kaon 496 MeV masses [13] we find the light and strange quark condensates respectively to be -42 MeV/fm^3 and -273 MeV/fm^3. The averaged vacuum contribution to the fields of dimension four in the equation of state can be calculated from the operator product expansion [12] using $< G^2 >_0 + m_q < \bar{q}q >_0$. Thus we find the estimated vacuum contributions to these two extreme cases to be 1.91 GeV/fm^3 for the light quarks and 1.68 GeV/fm^3 for the strange quarks. Furthermore, we remark here that both the gluon and quark condensates have the same color singlet ground state 0^{++} often associated with the scalar glueball state [14].

For the present work we have chosen these extreme cases for special states of baryonic structure. If we look at the spin 3/2 structure of Δ and Ω^- as examples of these ground state structures, we get a minimal effect from the spin entanglement and flavor mixing. Nevertheless, we include the degeneracy factor due to the quark spins for the integration over the momenta. The usual sum over the flavors is replaced by a factor of three in the baryons. We keep the degeneracy factor due to the gluon spins since both polarizations are possible. In the following work we shall use the trace anomaly for the substitution of the above extracted values for the vacuum condensates into the equation of state. It is known [9,10,11] that the temperature has very little effect on these values at temperatures well below 100 MeV. Thus we can look at the quantum effects of the entropy on the bag constant $\mathbf{B}$ for low temperatures T and small chemical potentials μ_q. Here we do not look at the explicit dependence of the bag constant on μ_q at zero temperature.

2 Equation of State

At finite temperatures and quark chemical potentials we choose the grand canonical partition function $\mathcal{Z}(T, V, \mu_q)$ in order to write down the equation of state in terms of difference between the energy density and the pressure $\varepsilon(T, \mu_q) - 3p(T, \mu_q)$. This expression may be represented in terms of the trace of the energy-momentum tensor averaged over these variables $\langle \Theta^\nu_\nu \rangle_{T,\mu_q}$, where the repeated indices represent the sum over the Lorentz indices. In the formulation of the bag model [6] the thermodynamics is usually included with a bag energy density $\varepsilon = +\mathbf{B}$ and a bag pressure $p = -\mathbf{B}$, which generally represents the energy density and the confining pressure of the bag against the vacuum. Hereupon, we may describe the expectation values for the gluon and quark condensates from the equation of state as a limiting value which exists in the sense $\lim_{T\to 0} < \Theta^\nu_\nu >_T \equiv \epsilon - 3p$. After applying the first law of thermodynamics relating the internal energy density to the entropy density, the pressure and the chemical potential we find the equation of state using the proton volume V_P from the charge radius [13] with the value 2.76 fm^3.

$$\langle \Theta^\nu_\nu \rangle_{T,\mu_q} = T \left[\frac{3\mathcal{S}_{q,3}(T)}{V_P} + s(T, \mu_q) \right] - 4 \left[p(T, \mu_q) - \mathbf{B} \right] + 3 \, \mu_q \, \rho(T, \mu_q) \, . \tag{2}$$

Here we include [4, 5] the ground state entropy $\mathcal{S}_{q,3}$ and the bag constant $\mathbf{B}$, which is usually assumed as independent of the parameters of the ensemble. The other thermodynamical quantities in the equation of state are given as follows:

$$p(T, \mu_q) = \frac{3}{\pi^2} T \int_0^\infty k^2 \, dk \left\{ \ln \left[1 + e^{-\frac{\epsilon(k)+\mu_q}{T}} \right] + \ln \left[1 + e^{-\frac{\epsilon(k)-\mu_q}{T}} \right] \right\} + \frac{8\pi^2}{45} T^4, \tag{3}$$

$$\rho(T, \mu_q) = \frac{3}{\pi^2} \int_0^\infty k^2 \, dk \left\{ \frac{1}{e^{\frac{\epsilon(k)-\mu_q}{T}} + 1} - \frac{1}{e^{\frac{\epsilon(k)+\mu_q}{T}} + 1} \right\}, \tag{4}$$

$$s(T, \mu_q) = \frac{3}{\pi^2} \frac{1}{T} \int_0^\infty k^2 \, dk \left\{ \frac{\epsilon(k)+\mu_q}{e^{\frac{\epsilon(k)+\mu_q}{T}} + 1} + \frac{\epsilon(k)-\mu_q}{e^{\frac{\epsilon(k)-\mu_q}{T}} + 1} \right\} + \frac{p_q(T, \mu_q)}{T} + \frac{32\pi^2}{45} T^3, \tag{5}$$

$$\mathcal{S}_{q,3}(T) = -\frac{1}{3} \left(1 - e^{-m_q/T} \right) \ln \left[\frac{1}{3} \left(1 - e^{-m_q/T} \right) \right] - 2z \left[\ln(z) \cos(\theta) - \theta \sin(\theta) \right], \tag{6}$$

where $\varepsilon(k)^2 = m_q^2 + k^2$ is the single particle energy and the values for z and θ respectively are

$$
\begin{aligned}
z &= \frac{1}{3}\left[1 + e^{-m_q/T} \;+\; e^{-2\,m_q/T}\right]^{1/2} \;, \\
\theta &= \arctan\left(\frac{\sqrt{3}\, e^{-m_q/T}}{2 + e^{-m_q/T}}\right) .
\end{aligned} \tag{7}
$$

The equations 3, 4 and 5 give the pressure, the baryon number density and the entropy density inside the baryonic bag depending on T and μ_q in the usual way. In Eq. 5 the expression $p_q(T, \mu_q)$ means just the quark contribution to the pressure in Eq. 3. The effective degrees of freedom which we are considering here are merely the quark colors. The contributions of gluons to these quantities are also taken into account, for which we considered the spin degeneracy due to the possible polarization. However, for the sake of simplicity, we have given the gluon radiation in the grand canonical partition function in an approximated form[1] which, for instance, appears in the second term of Eq. 3. This approximation is appropriate since both T and μ_q remain small compared to Λ_{QCD}.

By using the values of vacuum condensates given above, we can numerically solve Eq. 2 to get the dependence of the bag constant $\mathbf{B}$ upon the temperature T and quark chemical potential μ_q. Here we have considered two special baryonic structures of the hadron bag model, for which we shall compare the constancy of the bag constant with and without the quantum entropy $\mathcal{S}_{q,3}$. The inclusion of quantum entropy term [4, 5] given in Eq. 6 when set into the equation of state Eq. 2 has the effect of decreasing the pressure inside the bag. This leads to a decrease in the value of $\mathbf{B}$ needed to preserve the hadron bag's stability against the force of the outside vacuum.

3 Results and Discussion

In Fig. 1(a) we plot $4\mathbf{B}$ as a function of both T and μ_q for the baryonic state Δ given in Eq. 2 while leaving out the quantum entropy contribution [4, 5]. Here we have used the vacuum expectation value 1.91 GeV/fm^3 for the light quarks. We notice - at least for the region of T and μ_q which we are considering in Fig. 1 up to 50 MeV - that $4\mathbf{B}$ almost stays constant at the very low temperatures. Afterwards it only slightly increases with T. However, the dependence on μ_q is in comparison quite weak. For relatively large μ_q and very small temperature $4\mathbf{B}$ remains constant. For increasing values of μ_q the size of $4\mathbf{B}$ decreases. For larger T the decrease due to μ_q will be moderated.

In Fig. 1(b) we show the corresponding results for he baryonic state Ω without the quantum entropy. Here we have used the vacuum expectation value 1.68 GeV/fm^3 for the strange quarks. In this case we note the range of both T and μ_q in which $4\mathbf{B}$ remains constant is much wider than in the case of Δ. This fact is due to the strange quark mass, which is very much heavier than the masses of light quarks. Beyond this range it

[1] At high temperatures the pressure of the equilibrated ideal bosonic gas of gluons reads

$$
p(T)_{gluon} \;=\; \frac{8}{45}\pi^2\, T^4 \left[1 - \frac{15\alpha_s}{4\pi} + \cdots\right]
$$

where α_s is the running strong interaction coupling constant, which depends on T and μ_q. At low temperatures and small chemical potentials we can take $\alpha_s \to 0$.

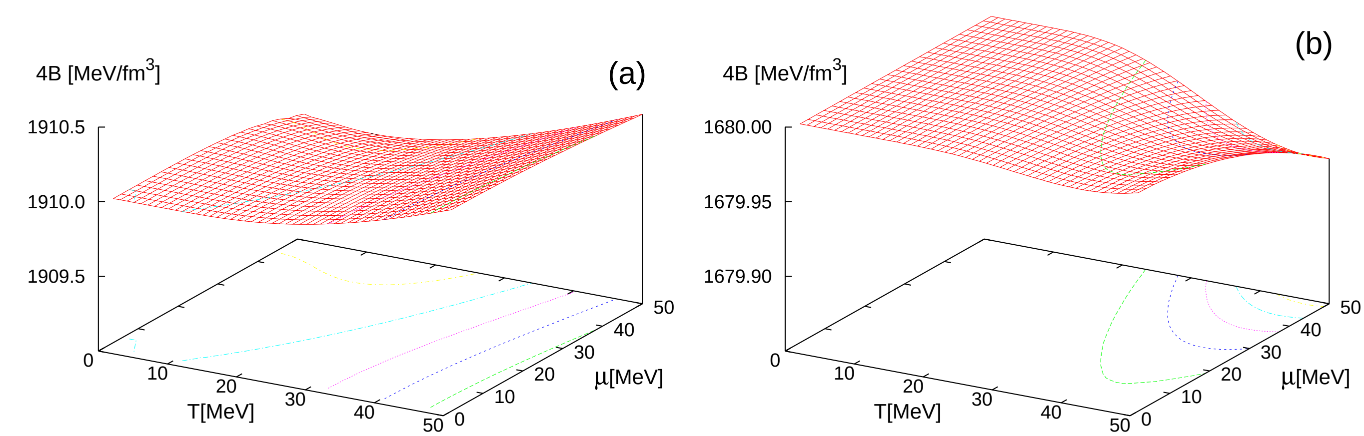

Figure 1: The panel (a) depicts $4B$ as a function of T and μ_q in a system with three light quarks. $\mathcal{S}(T)_{q,3}$ is not included here in the equation of state. The panel (b) gives the same dependence for three strange quarks.

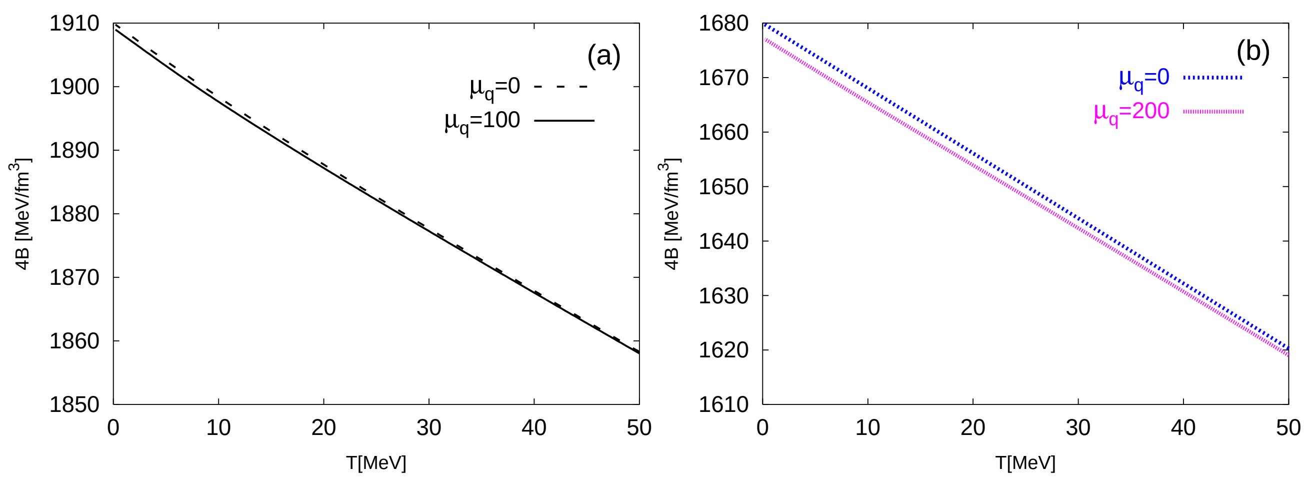

Figure 2: The panel (a) shows the thermal structure of three light quarks in a hadron bag for two values of the chemical potential. The similar thermal structure for three strange quarks in a hadron bag appears in (b). Both have the added quantum entropy term $\mathcal{S}(T)_{q,3}$.

decreases only marginally. Thereafter it starts to increase in a region not shown here. For larger T and μ_q we could expect that $4\mathbf{B}$ in the Ω bag should increase in a way similar to the Δ bag. Thus we may conclude that $4\mathbf{B}$ is almost constant, especially at very low temperatures and very small quark chemical potentials, when we do not consider the quantum entropy in the equation of state. Similarly we note that the dependence on μ_q is small but not fully negligible even though at higher values of T it seems not depending on μ_q. In the high temperature region we expect that the values of $\epsilon(T) - 3p(T)$ do not vanish for which the gluonic radiation contributions become very dominant. Thus at vanishing temperatures $\epsilon - 3p$ approaches $4B$. We may consider the region of small T and μ_q in which the bag constant remains relatively unchanged, which for the strange quarks is much wider.

The inclusion of the quantum entropy density leads to quite different qualitative effects which may be seen in Fig. 2. Here we note an almost linear decrease of $4\mathbf{B}$ with increasing T. The dependence on both T and μ_q is in Ω bags stronger than that in Δ bags. We should emphasize that the changes in $4\mathbf{B}$ from μ_q is a gradually varying one with a decrease in the slope. This is quite different from T dependence, for which $4\mathbf{B}$ is almost linearly decreasing function. One reason for the rather weak dependence on μ_q is that the quantum entropy term, $\mathcal{S}$, which is not included in the grand canonical partition function, has only been given a dependence on T. We may expect that this behavior will be valid for a much wider range of temperature, perhaps even up to T_c.

4 Conclusion and Outlook

Finally we are able to determine that the presence of the quantum entropy arising from the $SU(3)_c$ color symmetry in the equation of state provides a strong temperature dependence for the bag constant $\mathbf{B}$. However, the equation of state yields a much weaker dependence of $\mathbf{B}$ on the quark chemical potential μ_q, from which $\mathbf{B}$ is changed very slowly at low temperatures. The inclusion of the quantum entropy term in the equation of state for the two considered baryonic structures leads to an acceleration of the decrease in $\mathbf{B}$ with increasing temperatures has an approximately linear decline. We have contrasted these results in both these cases to the same properties without the quantum entropy.

Based on these results we shall plan further studies into the behavior of the structure of confined quark matter at low temperatures and small quark chemical potentials. A further consideration of the idea that glueballs could appear as 0^{++} state in a Bose-Einstein condensate [14] seems to be a quite promising further point. The importance of the glueball degrees of freedom for describing the hadronic phase for temperature below T_c has already been investigated [15, 16]. Also the existence of spin-color waves in the bag we would like to study further.

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